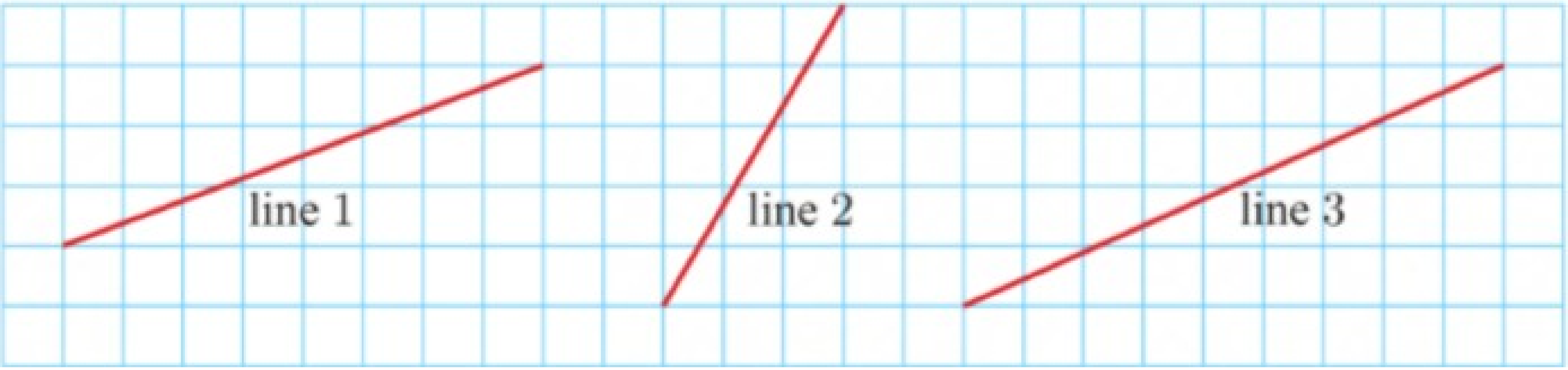
C

GRADIENT

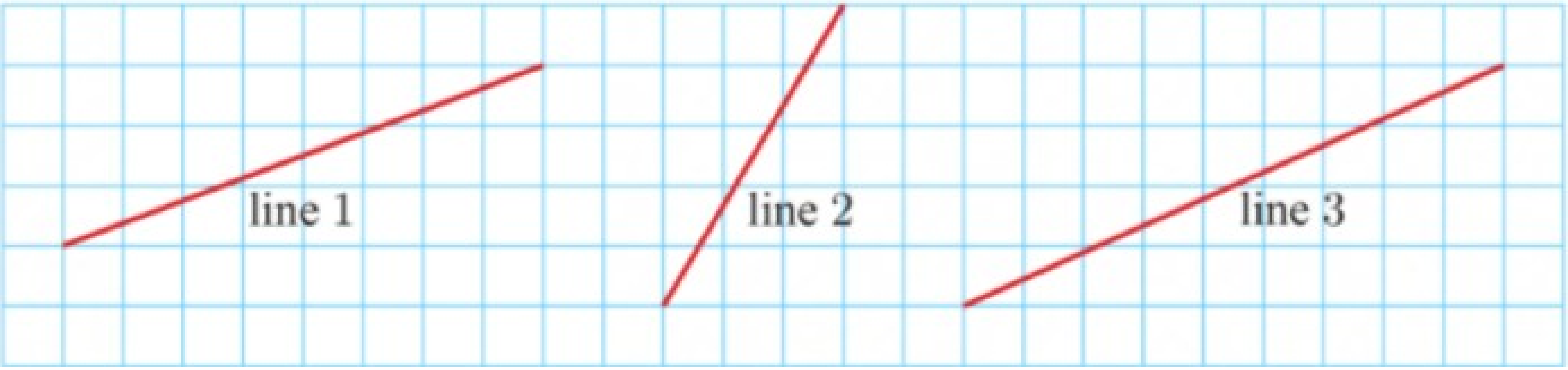
C GRADIENT

Consider the lines shown:



C GRADIENT

Consider the lines shown:



We can see that line 2 rises much faster than the other two lines, so line 2 is steepest.

However, most people would find it hard to tell which of lines 1 and 3 is steeper just by looking at them. We therefore need a more precise way to measure the steepness of a line.

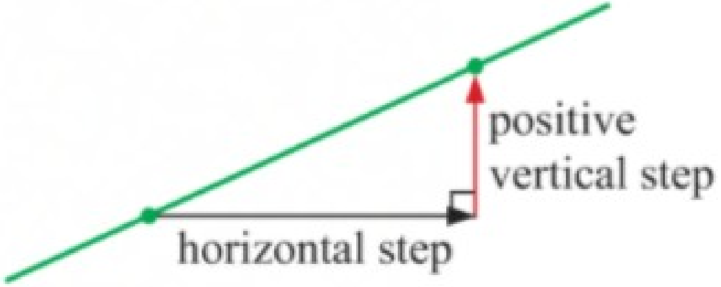
The **gradient** of a line is a measure of its steepness.

To calculate the gradient of a line, we first choose any two distinct points on the line. We can move from one point to the other by making a positive **horizontal step** followed by a **vertical step**.

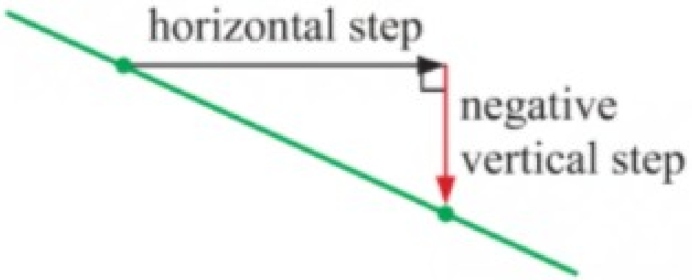
The gradient is calculated by dividing the vertical step by the horizontal step.

$$\text{The } \textbf{gradient} \text{ of a line} = \frac{\text{vertical step}}{\text{horizontal step}} \quad \text{or} \quad \frac{y\text{-step}}{x\text{-step}}.$$

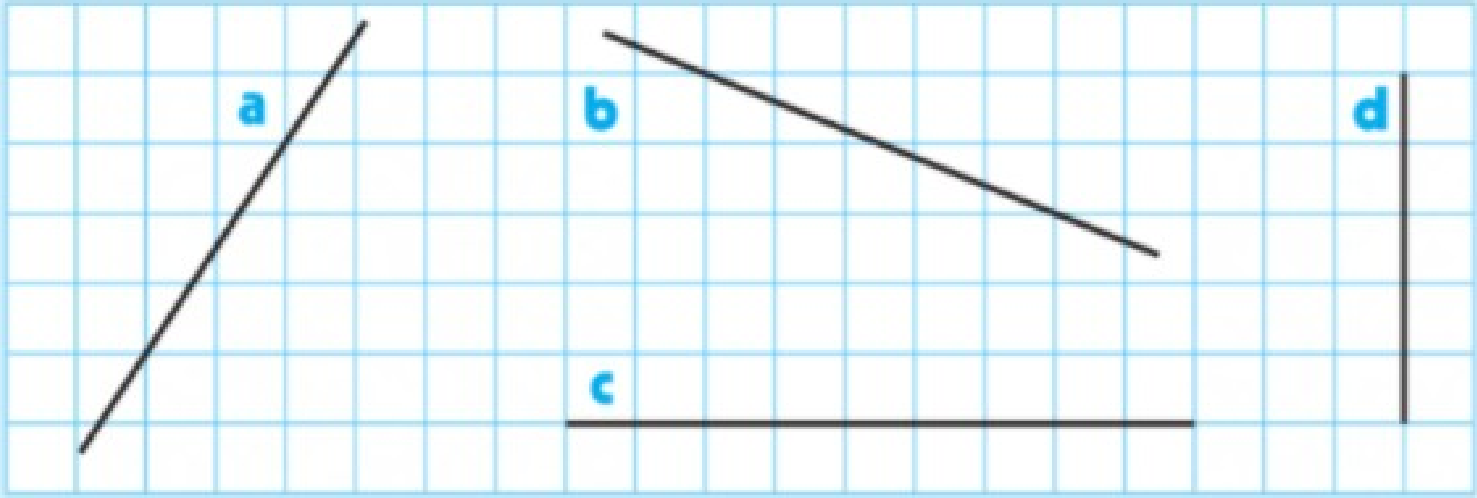
If the line is sloping upwards, then both steps are positive, so the line has a **positive gradient**.



If the line is sloping downwards, the horizontal step is positive and the vertical step is negative, so the line has a **negative gradient**.



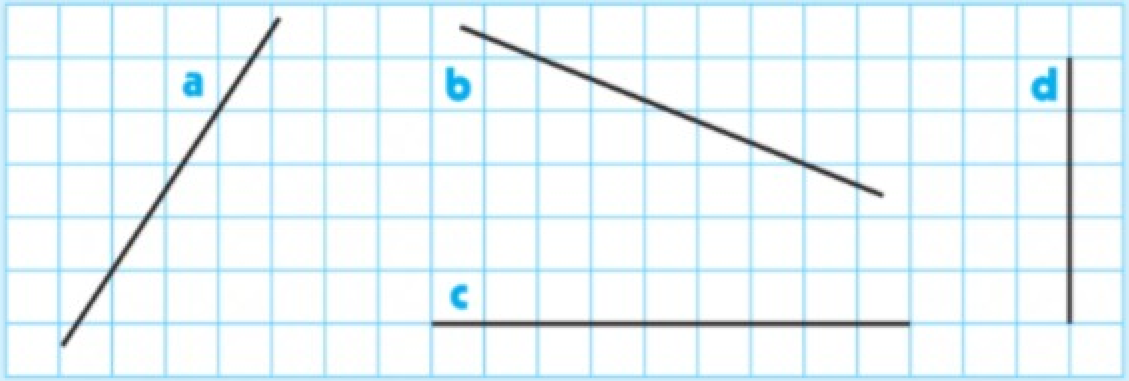
Find the gradient of each line segment:

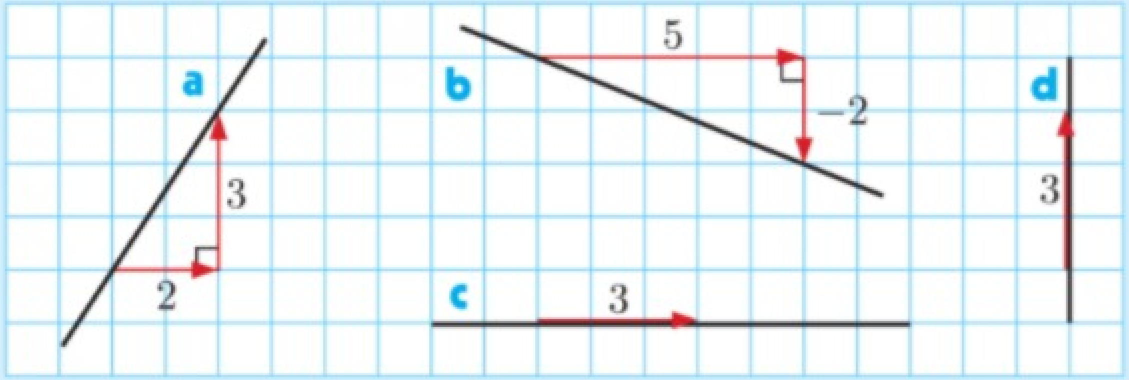


Example 7

Self Tutor

Find the gradient of each line segment:





a gradient $= \frac{3}{2}$

b gradient $= \frac{-2}{5} = -\frac{2}{5}$

c gradient $= \frac{0}{3} = 0$

d gradient $= \frac{3}{0}$ which is undefined

From the previous **Example**, we can see that:

- The gradient of all **horizontal** lines is **0**, since the vertical step is 0.
- The gradient of all **vertical** lines is **undefined**, since the horizontal step is 0.

Example 8 **Self Tutor**

Draw a line with gradient $-\frac{2}{3}$, through the point $(2, 4)$.

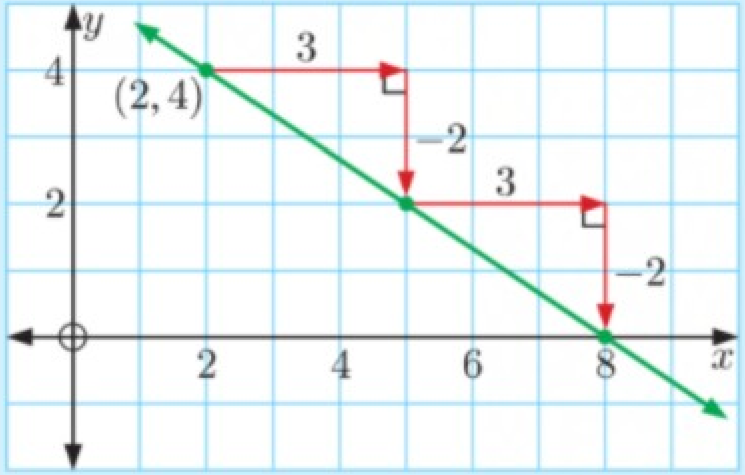
Example 8

Self Tutor

Draw a line with gradient $-\frac{2}{3}$, through the point $(2, 4)$.

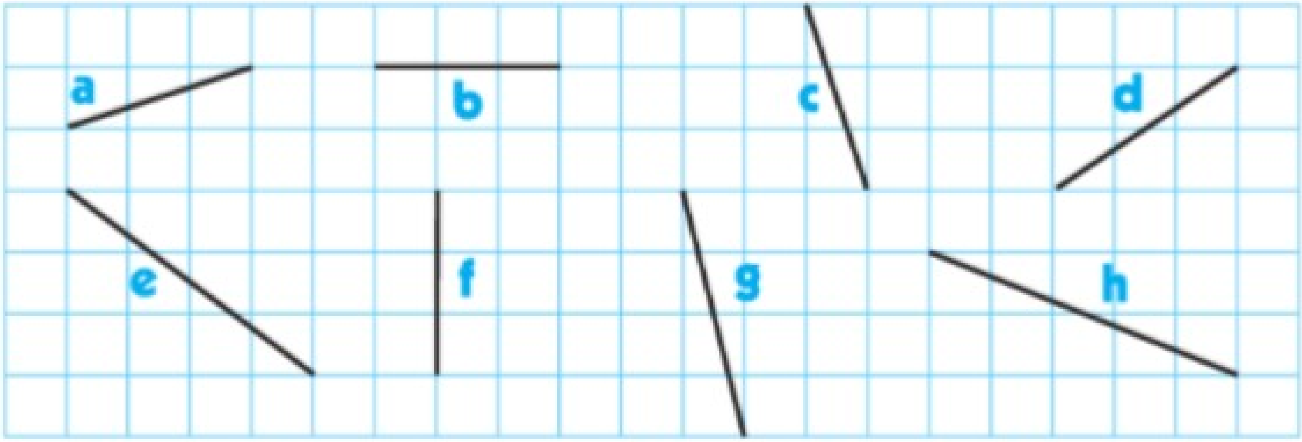
Plot the point $(2, 4)$.

$$\text{gradient} = -\frac{2}{3} = \frac{-2 \leftarrow y\text{-step}}{3 \leftarrow x\text{-step}}$$



Use a positive x-step.

1 Find the gradient of each line segment:



2 On grid paper, draw a line segment with gradient:

a $\frac{3}{4}$ **b** $-\frac{1}{2}$ **c** 2 **d** -3 **e** 0 **f** $-\frac{2}{5}$

3 Draw a line with gradient $\frac{1}{2}$, through the point $(3, -1)$.

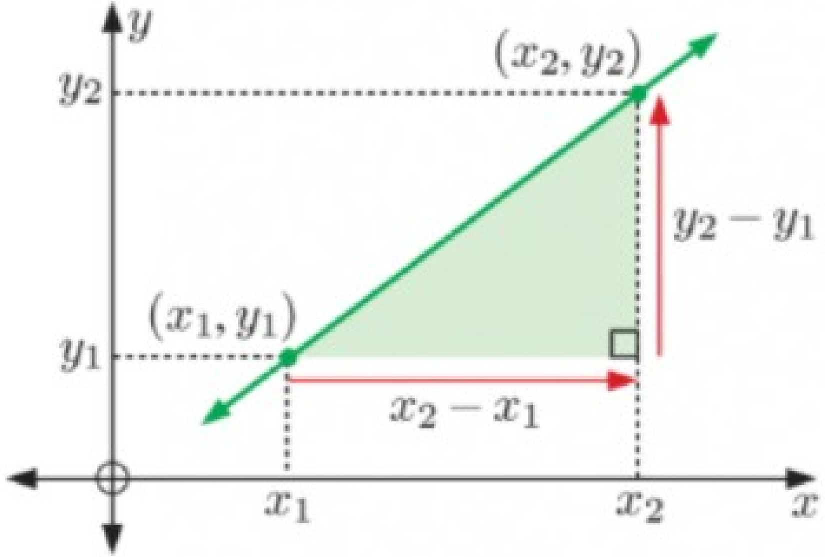
4 Draw a line with gradient $-\frac{3}{4}$, through the point $(-1, 3)$.

5 On the same set of axes, draw lines through $(2, 3)$ with gradients $\frac{1}{3}$, $\frac{3}{4}$, 2, and 4.

6 On the same set of axes, draw lines through $(-1, 2)$ with gradients 0, $-\frac{2}{5}$, -2, and -5.

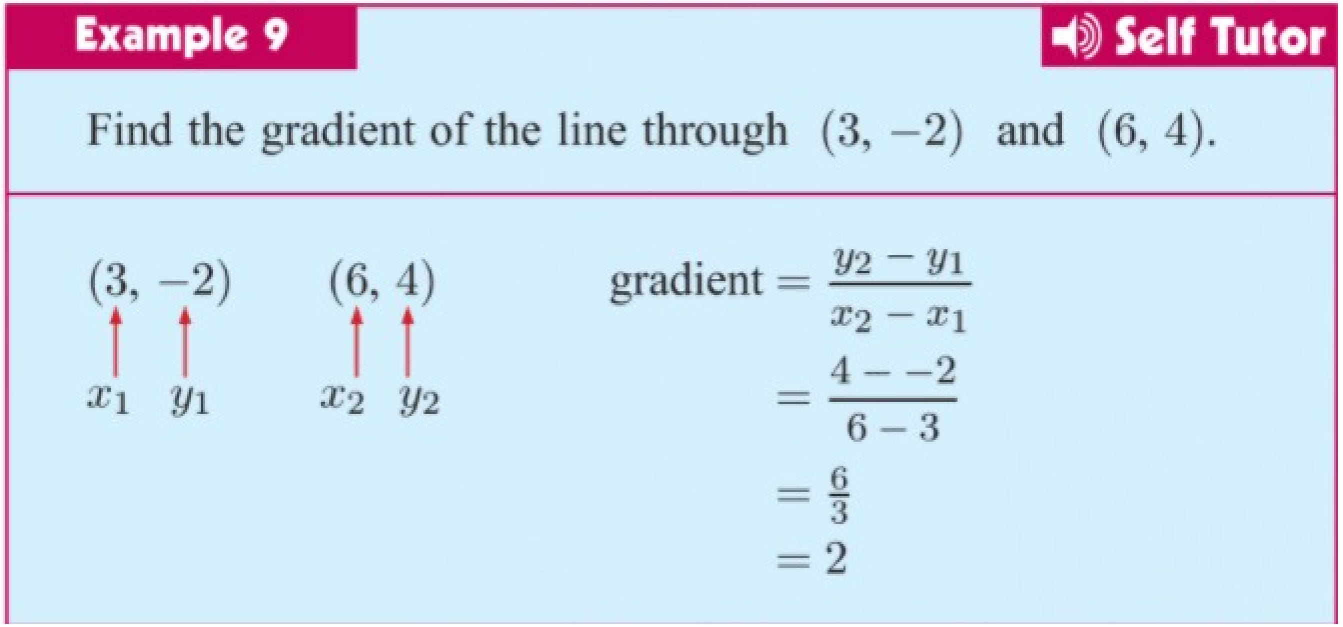
THE GRADIENT FORMULA

The **gradient** of the line through (x_1, y_1) and (x_2, y_2) is $\dfrac{y_2 - y_1}{x_2 - x_1}$.



Example 9 **Self Tutor**

Find the gradient of the line through $(3, -2)$ and $(6, 4)$.



Example 9

Self Tutor

Find the gradient of the line through $(3, -2)$ and $(6, 4)$.

$$(\underset{\underset{x_1}{\uparrow}}{3}, \underset{\underset{y_1}{\uparrow}}{-2}) \qquad (\underset{\underset{x_2}{\uparrow}}{6}, \underset{\underset{y_2}{\uparrow}}{4})$$

$$\begin{aligned} \text{gradient} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4 - -2}{6 - 3} \\ &= \tfrac{6}{3} \\ &= 2 \end{aligned}$$

EXERCISE 6C.2

1 Use the gradient formula to find the gradient of the line through A$(-2, -3)$ and B$(5, 1)$. Plot the line segment [AB] on a set of axes to illustrate your answer.

2 Find the gradient of the line segment joining:

- **a** $(2, 3)$ and $(7, 4)$
- **b** $(5, 7)$ and $(1, 6)$
- **c** $(1, -2)$ and $(3, 6)$
- **d** $(5, 5)$ and $(-1, 5)$
- **e** $(3, -1)$ and $(3, -4)$
- **f** $(5, -1)$ and $(-2, -3)$
- **g** $(-5, 2)$ and $(2, 0)$
- **h** $(0, -1)$ and $(-2, -3)$
- **i** $(-1, 7)$ and $(11, -9)$.

Example 10 **Self Tutor**

Find t given that the line segment joining $(5, -2)$ and $(9, t)$ has gradient $\frac{2}{3}$.

Example 10

Self Tutor

Find t given that the line segment joining $(5, -2)$ and $(9, t)$ has gradient $\frac{2}{3}$.

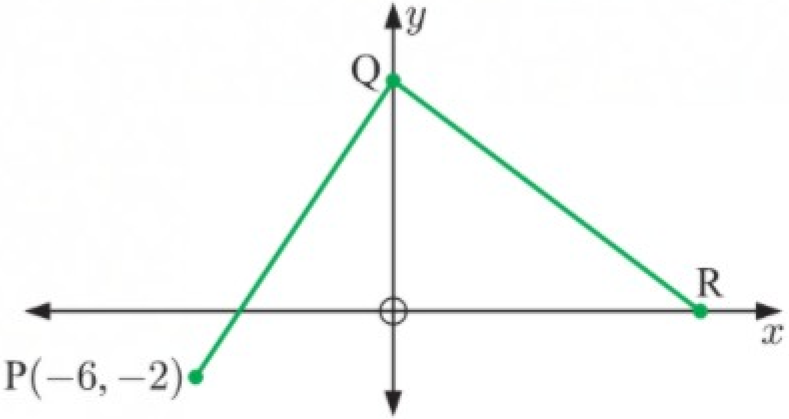
The line segment joining $(5, -2)$ and $(9, t)$ has gradient $= \dfrac{t - -2}{9 - 5} = \dfrac{t+2}{4}$.

$$\begin{aligned} \therefore\ \frac{t+2}{4} &= \frac{2}{3} \\ \therefore\ 3(t+2) &= 8 \\ \therefore\ 3t + 6 &= 8 \\ \therefore\ 3t &= 2 \\ \therefore\ t &= \tfrac{2}{3} \end{aligned}$$

3 Find t given that the line segment joining:

- **a** $(-3, 5)$ and $(4, t)$ has gradient 2
- **b** $(5, t)$ and $(10, 12)$ has gradient $-\frac{1}{2}$
- **c** $(3, -6)$ and $(t, -2)$ has gradient 3
- **d** $(t, 9)$ and $(4, 7)$ has gradient $-\frac{3}{5}$
- **e** $(2, 5)$ and (t, t) has gradient $\frac{4}{7}$
- **f** $(t, 2t)$ and $(-3, 12)$ has gradient $-\frac{1}{4}$.

4



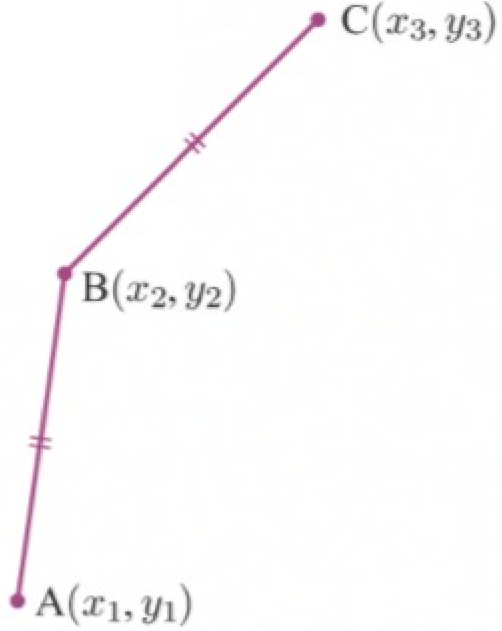
The gradient of [PQ] is $\frac{3}{2}$, and the gradient of [QR] is $-\frac{3}{4}$. Find the coordinates of R.

5 $A(x_1, y_1)$, $B(x_2, y_2)$, and $C(x_3, y_3)$ are three points such that the gradient of [AB] is 7, the gradient of [BC] is 1, and $AB = BC$.

a Use the gradient formula to show that
$(y_2 - y_1)^2 = 49(x_2 - x_1)^2$ and
$(y_3 - y_2)^2 = (x_3 - x_2)^2$.

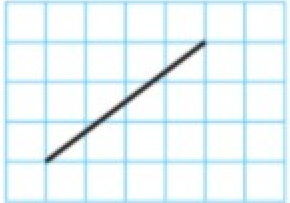
b Use **a** and the distance formula to show that
$x_3 - x_2 = 5(x_2 - x_1)$.

c Hence, find the gradient of [AC].

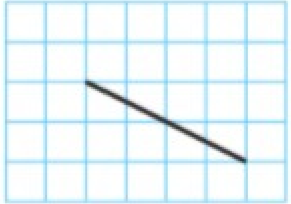


1 **a** $\frac{1}{3}$ **b** 0 **c** -3 **d** $\frac{2}{3}$ **e** $-\frac{3}{4}$
f undefined **g** -4 **h** $-\frac{2}{5}$

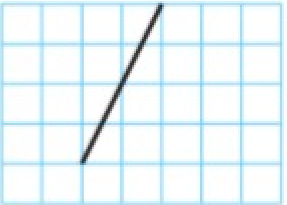
2 **a**



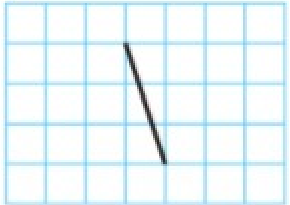
b



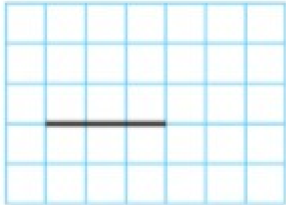
c



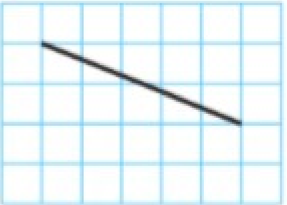
d



e



f



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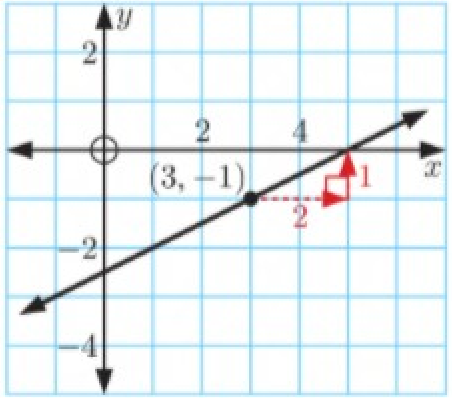

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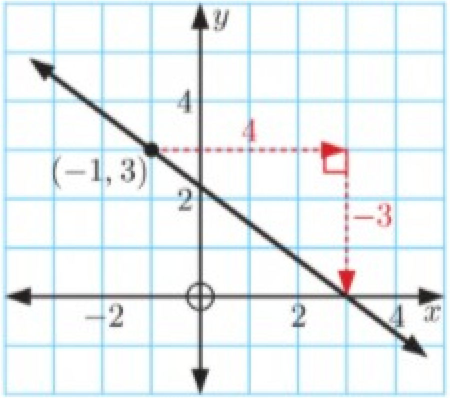

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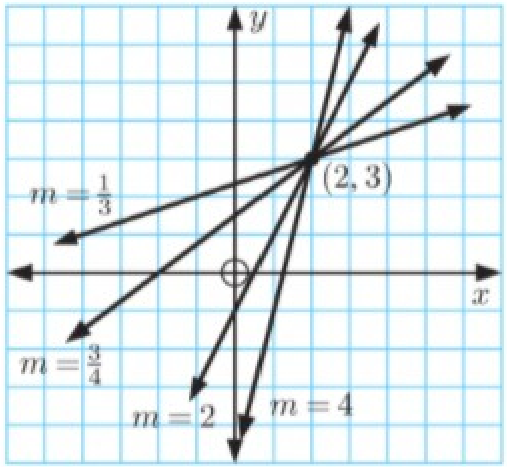

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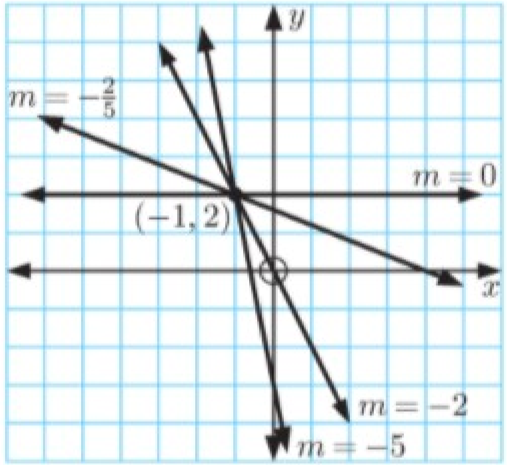
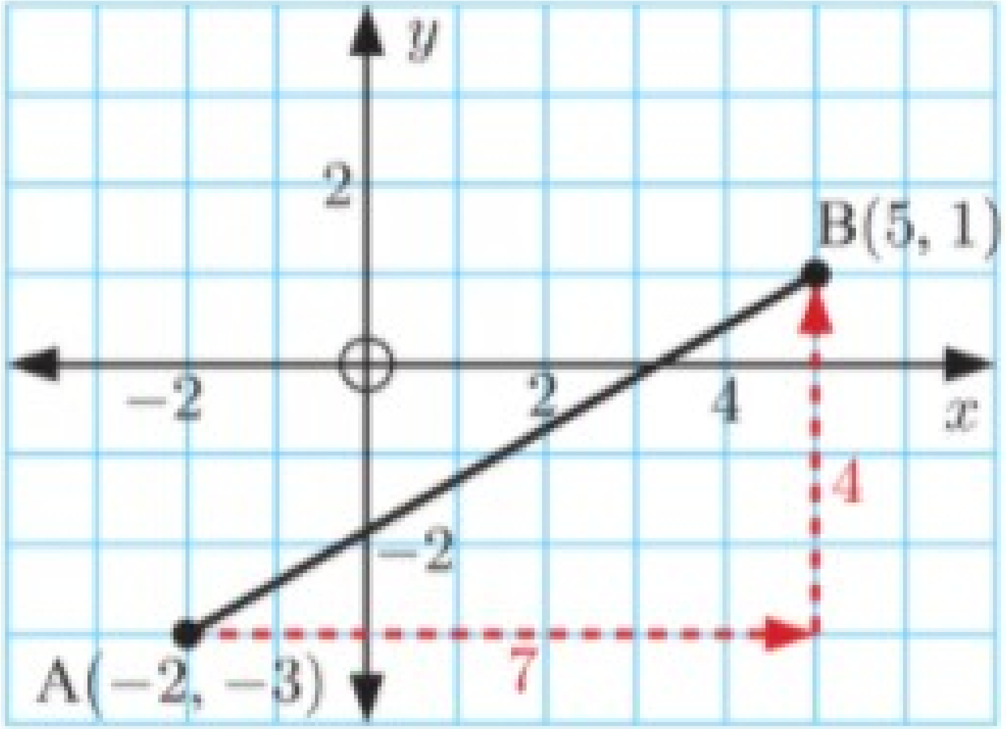


EXERCISE 6C.2

1 $\frac{4}{7}$



2 **a** $\frac{1}{5}$ **b** $\frac{1}{4}$ **c** 4 **d** 0 **e** undefined
f $\frac{2}{7}$ **g** $-\frac{2}{7}$ **h** 1 **i** $-\frac{4}{3}$

3 **a** $t = 19$ **b** $t = \frac{29}{2}$ **c** $t = \frac{13}{3}$ **d** $t = \frac{2}{3}$
e $t = 9$ **f** $t = 5$

4 $\mathrm{Q}(0, 7)$, $\mathrm{R}(\frac{28}{3}, 0)$

5 **c** gradient of [AC] $= 2$